Asymmetric Normalization Aided Information Diffusion for Socially-Aware Mobile Networks

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***Abstract*—How to improve the information diffusion coverage rate in socially-aware mobile networks has drawn great attention. To address this issue, the concept of the tie strength, the partial strength and the value strength were proposed in order to achieve a superior criterion for information diffusion. However, the previous works did not consider the existence of various patterns among the nodes in socially-aware mobile networks. In this paper, we propose the asymmetric normalization forms of the partial strength as well as the value strength. Moreover, we explore the essence of asymmetry and its influence on information diffusion relying on analyzing the characteristics of graph structures as well as information local traps. Simulation results on the real-world social network and on the mobile network verify that our proposed asymmetric normalization forms are beneficial to promoting information diffusion.**

I. Introduction

The past decade witnessed the booming of social networks and a range of research topics had been studied [1], such as viral marketing, public opinion control, online advertising, etc. Socially-aware networks provide users with an information sharing platform, where we are capable of sharing what we have seen or what we have heard through the specific communication links [2]. Essentially, mobile networks are driven by users, where the social attribute among these users may improve the performance of the network [3]. Hence, the socially-aware mobile network is proposed in order to explore the "social factors" of networks as well as to probe the role they may play in the information diffusion [4] [5].

It is of great significance for the socially-aware mobile networks to explore the information diffusion. Numerous scenarios in socially-aware mobile networks, including public opinion information control and viral marketing, are directly related to information coverage ratio, which is calculated by the ratio of the entities that have received information versus the total number of the entities in the network. Specifically, relying on the efficient control of the key nodes and links, we are capable of facilitating useful information transmission and preventing malicious information broadcasting in the public opinion information control [6]. Moreover, a profitable individual forwarding or content recommendation scheme is beneficial in terms of improving the coverage ratio in the viral marketing [1].

The state of the art about the information diffusion mechanisms can be classified into three categories: classical flooding mechanism, topological property based centralized mechanism and topological property based distributed mechanism. The study of information diffusion originates from the propagation dynamics in complex networks. Several classic virus propagation models, such as SI, SIS and SIR, are still been used in the study of information diffusion [7] [8]. Even aided by some mathematical optimization methods, such as evolutionary game [9] and Markov decision process [10], these mechanisms suffer a large network load with no regard to the network topology. Based on these models, Kitsak *et al.* [7] explored the influential spreaders in the network for ensuring a more efficient information diffusion. The information diffusion mechanisms of the second category consider some topological properties of the network, such as betweenness centrality [11], eigenvector centrality [12] and K-shell [7], to achieve a better information diffusion. Wang *et al.* [11] analyzed the role of the betweenness centrality in the information diffusion and selected the nodes with high betweenness values as relay nodes, which improved the information coverage ratio at a large degree. Banerjee *et al.* [12] found that eigenvector centrality played a critical role in the diffusion of microfinance. Their results showed that the injection points with a higher eigenvector centrality usrally had a higher microfinance participation. However, these schemes require a network-wide topology and are not suitable for dynamic networks having varying topologies, while the majority of real-life networks are dynamic, such as MANETs, VANETs, etc. Therefore, other mechanism of information diffusion over the dynamic graph structure is needed to meet the realistic engineering. In contrast to the second type, topological property based distributed mechanisms were proposed in order to analyze the dynamic networks. In [13], Granovetter *et al.* proposed the tie strength and explained the social phenomenon that most of our jobs were recommended by the people who we were not familiar with. Zhao *et al.* [14] further verified the roles of weak tie strength based on two online social networks and pointed out that the information local trap[1] might weaken the the performance of weak tie. Onnela *et al.* [15] discovered the coupling relationship between tie strengths and

[1]The information local trap, i.e., the early-termination of information diffusion, refers to the phenomenon that the information is unable to be transferred, which means that all the surrounding nodes have forwarded the information.

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the local structure of networks. Moreover, other forms of strength definition were provided, such as the partial strength as well as the value strength [11], for obtaining optimal information diffusion under certain scenario. However, most of the existing works focused more on the static networks and did not consider the two patterns, i.e., the asymmetric and symmetric way[2], among the nodes and did not give the readers the intuitive theoretical explanation.

Inspired by the aforementioned problems, in this paper, we propose the asymmetric normalization forms of the partial strength as well as the value strength. Moreover, we explain the essence of asymmetry and its influence relying on analyzing the characteristics of graph structures as well as information local traps with the following original contributions.

- Asymmetric normalization forms: Based on the concept of the partial strength and the value strength, we propose their corresponding asymmetric normalization forms and demonstrate their superior performance on the information diffusion for socially-aware mobile networks.
- Asymmetry analysis: Both the network structural characteristics and the forwarding nodes selection scheme play a critical role in information diffusion, where the information local traps may become a key contributor.
- Data-driven experiments: We use the real-world dataset, i.e., the Flickr and the CA-GrQc [16], in order to verify the performance of information diffusion over socially-aware mobile networks. Furthermore, the information diffusion in mobile networks is conducted and analyzed on a mobile D2D network.

The remainders are outlined as follows. The information propagation models and several strengths in a socially-aware network are introduced in Section II. Relying on different strengths, the information diffusion performances are discussed in Section III and IV. In Section V, we clarify the essence of asymmetric strengths in terms of their superior performances, followed by our conclusions in Section VI.

II. Information Propagation Models and Asymmetric Normalization Strengths

In this section, we commence with introducing the cascade information propagation model (Section II-A). Then, we present a range of different definitions of symmetric strengths (Section II-B), and their asymmetric normalization forms (Section II-C).

A. Cascade Information Propagation Model

In this subsection, the cascade information propagation model is presented in order to formulate the information diffusion process. The information diffusion starts from the source node. Then, R_i relay nodes will be selected in terms of a certain transition probability from the source forwarding node's neighbors. Once the node becomes a forwarding node, its neighbors will be informed. Specifically, the initial state of all the nodes are denoted as I_0, which means that it has never received/forwarded the information, i.e., uninformed state. I_1 represents the node that has received specific information. The state, namely I_2, represents that the node has forwarded the information. The specific cascade information propagation model can be described as follows.

1) When $t = 0$, select a node in the network as the source node, i.e., i, and mark its state as I_1.
2) When $t = t+1$, the state of all the neighbor nodes of i are changed to I_1, where the number of i's neighbor nodes is denoted as K_i. Then, add the node i into the retired set Ω, and change its state into I_2, which indicates that the node i has finished forwarding the information.
3) Calculate the number of the next-time forwarding nodes, i.e., $R_i = \lceil \beta K_i \rceil$, where β is the diffusion discount factor between 0 and 1, and $\lceil \cdot \rceil$ represents the rounding function.
4) Select the forwarding nodes whose state are I_0 or I_1 by the following transition probability:
$$P_{ij} = \frac{S_{ij}}{\sum\limits_{k=1}^{K_i} S_{ik}}, \tag{1}$$
where j or k is one of i's neighbors. S_{ij} represents the strength value of node j to i. If the node j is selected, add the node j to the forwarding nodes set, namely W. Repeat this process R_i times.
5) For each node in W, repeat the second step until W is empty, which is called as early termination due to local trap, or the time step reaches the upper time limit.

B. Symmetric Strength Aided Information Diffusion

Relying on the subsection II-A, we are capable of deducing that the critical part of the information diffusion models is the definition of the strength values in Eq. (1), i.e., S_{ij}. We denote the S_R, S_T, S_{SP}, S_{SV}, S_{AP}, S_{AV} to represent the random strength, tie strength, symmetric partial strength, symmetric value strength, asymmetric partial strength and asymmetric value strength, respectively.

Here, we present the notation which will be adopted later. The C_a means the neighborhood set of node a, the same with C_b. The intersection set $C_a \bigcap C_b$ denotes the common friends of the node a and b, and the union set $C_a \bigcup C_b$ means the whole friends of the node a and b. Moreover, the function $|\bullet|$ represents the total number of the elements in a set. And the difference set $C_b \backslash C_a$ reflects that the nodes who are the neighbors of the node b but not of a.

The random strength means selecting the relay nodes randomly. $S_R(a \to b)$ represents the random strength from the node a to node b, which can be given by $S_R(a \to b) = S_R(b \to a) = rand(1)$. The $rand(1)$ represents a random number between 0 and 1.

The strong tie strength can be denoted as $S_T(a \to b) = S_T(b \to a) = |C_a \bigcap C_b| \, / \, |C_a \bigcup C_b|$. Strong tie strength

[2] The symmetric forms focus their attention on the number of new neighbors that have not received the message, i.e., how to increase the information coverage ratio while they ignore how to avoid getting caught into the local trap and to increase the probability that message may continue to propagate. The asymmetric forms adjust the normalization methods relying on the different strength definitions, considering both the information coverage ratio as well as the local trap.

aided information diffusion shows that more mutual friends between node a and node b suggest more choices for node b or a to select a or b relay. In contrast to the strong tie strength, the weak tie strength can be denoted as $S_T(a \to b) = S_T(b \to a) = (|C_a \bigcap C_b|/|C_a \bigcup C_b|)^{-1}$. The weak tie strength means that the more common friends the node a and node b have, the more difficult the node b or a will be selected. However, the networks are often heterogeneous in real-life world, which show the characteristic of a power-law distribution. The denominators of above two tie strength are the intersection (common friends) of two sets, which may be affected by their heterogeneity. Hence, the partial strength is proposed in order to reduce the impact of heterogeneity.

To address the above issue, the partial strength and value strength were proposed by Wang *et al.* The strong partial strength is defined as $S_{SP}(a \to b) = (|C_a \bigcap C_b| + 1)/|C_b|$ and $S_{SP}(b \to a) = (|C_a \bigcap C_b| + 1)/|C_a|$. The $S_{SP}(a \to b)$ represents the partial strength value of the node a to the node b, and $S_{SP}(b \to a)$ denotes the partial strength value of the node b to a. As a contrast, the weak partial strength can be rewritten as $S_{SP}(a \to b) = ((|C_a \bigcap C_b| + 1)/|C_b|)^{-1}$ and $S_{SP}(b \to a) = ((|C_a \bigcap C_b| + 1)/|C_a|)^{-1}$. The strong value strength is defined as $S_{SV}(a \to b) = |C_b \backslash C_a|/|C_a|$ and $S_{SV}(b \to a) = |C_a \backslash C_b|/|C_b|$. The correspondingly opposite form of the strong value strength can be given by $S_{SV}(a \to b) = (|C_b \backslash C_a|/|C_a|)^{-1}$ and $S_{SV}(b \to a) = (|C_a \backslash C_b|/|C_b|)^{-1}$.

C. Asymmetric Strength Aided Information Diffusion

Based on the definition of the partial strength and the value strength, we find that the denominator of these strengths is the next-hop node or the source node, which means there exist two forms, i.e., the asymmetric and symmetric pattern.

1) Asymmetric Strong Partial Strength:

$$S_{AP}(a \to b) = \frac{|C_a \bigcap C_b| + 1}{|C_a|}, \tag{2}$$

$$S_{AP}(b \to a) = \frac{|C_a \bigcap C_b| + 1}{|C_b|}. \tag{3}$$

2) Asymmetric Weak Partial Strength:

$$S_{AP}(a \to b) = \left(\frac{|C_a \bigcap C_b| + 1}{|C_a|}\right)^{-1}, \tag{4}$$

$$S_{AP}(b \to a) = \left(\frac{|C_a \bigcap C_b| + 1}{|C_b|}\right)^{-1}. \tag{5}$$

3) Asymmetric Strong Value Strength:

$$S_{AV}(a \to b) = \frac{|C_b \backslash C_a|}{|C_b|}, \tag{6}$$

$$S_{AV}(b \to a) = \frac{|C_a \backslash C_b|}{|C_a|}. \tag{7}$$

4) Asymmetric Weak Value Strength:

$$S_{AV}(a \to b) = \left(\frac{|C_b \backslash C_a|}{|C_b|}\right)^{-1}, \tag{8}$$

$$S_{AV}(b \to a) = \left(\frac{|C_a \backslash C_b|}{|C_a|}\right)^{-1}. \tag{9}$$

The four algorithms above are the correspondingly asymmetric form of the previous algorithms in Section II-B, in essence, they are the different normalization form.

III. Information Diffusion Performances in Online Social Networks

In this section, we verify the asymmetric strength aided information diffusion algorithms in terms of the two real-world datasets, including the Flickr and CA-GrQc network [16]. The Flickr network is a large images sharing platform, while the CA-GrQc network is a collaboration about the general relativity in Arxiv. We commence with the characteristics analysis of the two datasets (Section IV). Then, the information diffusion performances are evaluated and analyzed (Section III-B).

A. Dataset Analysis

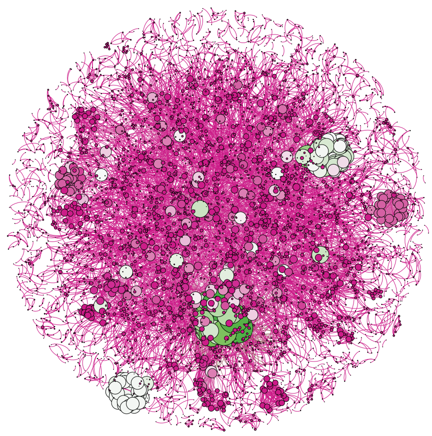
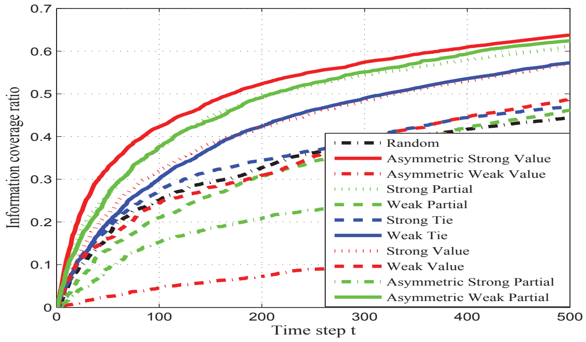


Fig. 1. The skeleton of the CA-GrQc dataset.

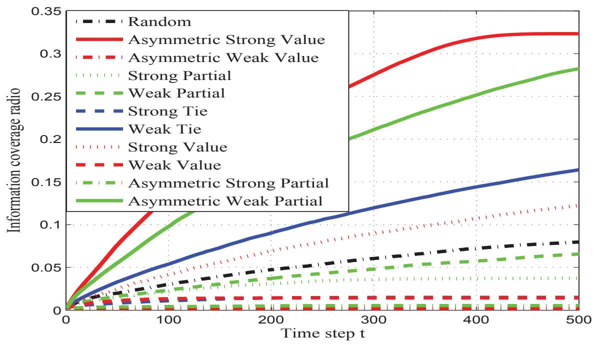
The Flickr dataset contains 80513 nodes and 5899882 edges. Furthermore, the edge represents the interaction between two users. As for the CA-GrQc dataset, as shown in Fig. 1, it includes 5242 nodes and 14496 edges. In the CA-GrQc, the node represents the researcher studying the general relativity and quantum cosmology, while the edge between two nodes indicates that the two researchers as least co-author one paper. Besides, the average degree of the Flickr and the Ca-GrQc is 146.557 and 5.526, respectively.

B. Information Diffusion Performances

In order to evaluate the performances of the information diffusion on socially-aware mobile networks, we design the following two experiments based on the cascade model. Furthermore, the information coverage ratio is used to measure the information diffusion performances. The experimental parameters are set as follows. In Flickr network, we first fix a single source and repeat 50 times experiments and calculate the average information coverage ratio for each strength. Moreover, we set the maximum time step as 500. The information diffusion process terminates when the time



(a) Information coverage ratio in Flickr network.



(b) Information coverage ratio in CA-GrQc network.

Fig. 2. Information coverage ratio in terms of different strengths aided forwarding nodes' selection scheme.

step arrives 500 or the early-termination occurs. In order to ensure a reasonable number of next hop nodes, we set the β to be 0.01 and 0.1 in Flickr and Ca-GrQc based on their the average degree. Fig. 2 (a) shows the average information coverage ratio in terms of different strengths aided schemes.

As for the CA-GrQc network, 10 information source nodes are selected randomly at the beginning. Similarly, we repeat 50 times experiments and calculate the average information coverage ratio for each strength. Also, the maximum time step is set to be 500. Simulation results are shown in Fig. 2 (b).

It is clear that the asymmetric strong value strength based scheme reaches the largest information coverage ratio with the increase of the time step. Moreover, the asymmetric weak partial based algorithm comes to the second versus others. Furthermore, our results match the famous sociological theory, i.e., weak tie plays a important role in bridging two groups, which could be explained via the comparison of the strong tie strength and the weak tie strength. The above results confirm that the asymmetric strength is beneficial in terms of the information diffusion. In addition, we discover that the two asymmetric normalization forms of each strength, i.e., asymmetric weak based strength and the asymmetric strong based strength, tend to present the polarization phenomenon. In other word, one form is with a very good performance, and the other is with an extremely poor performance.

IV. Information Diffusion Performances in Mobile Networks

As mentioned before, we have verified the superior performance of the asymmetric strength aided information diffusion mechanisms in online social networks. In this section, we continue to evaluate them on two dynamic-topology networks, i.e., a D2D network as well as a real-world vehicular network. Here, we assume that the nodes have forwarded the information before can forward again, in order to prevent the frequent early-termination of our mobile networks.

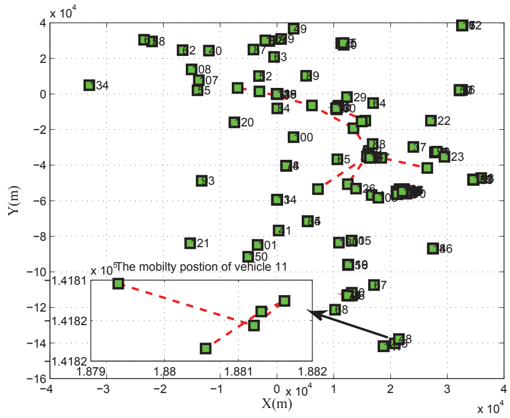


Fig. 3. The mobility model of the vehicular networks.

In this part, we adopt the mobility model proposed by Wang *et al.* [11]. The mobile network scenario includes 100 mobile entities, which are distributed in the range of 1000 m $\times$ 1000 m area initially. The node moves within their own circles with $r = 50$ m. Specifically, the deviation angle of each node follows the uniform distribution, i.e., $\theta \sim U(0, 2\pi)$, and its velocity obeys a Gaussian distribution, i.e., $\nu \sim \mathcal{N}(5, 1)$ m/s. The D2D communication range is set to be 100 m.

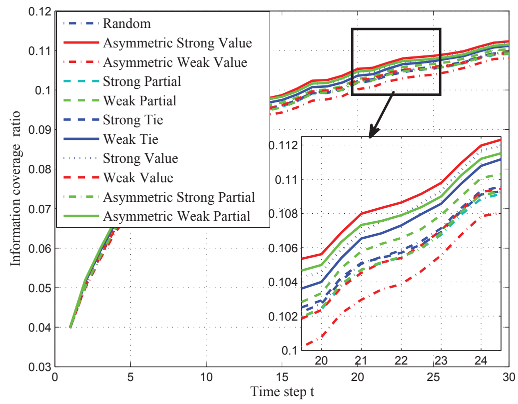


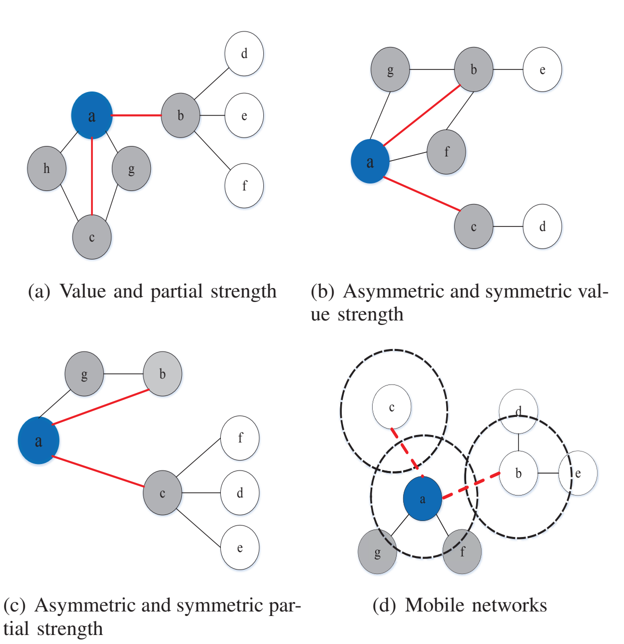
Fig. 4. Information coverage ratio in the D2D network.

Based on the mobility model mentioned above, we set the simulation interval time to be 10 s. Similarly, all the strength aided forwarding nodes' selection schemes are tested. Given the relatively small graph, we traverse all the nodes as the information source in different experiments. Moreover, for each strength, we run 10 times. Fig. 4 shows the average information coverage ratio in terms of different strengths aided forwarding nodes' selection schemes in our proposed D2D network. We can conclude that the asymmetric weak partial strength based- and asymmetric strong value strength based-forwarding nodes' selection schemes outperform others. Unlike the static online social networks, the mobility leads to a dynamic changes of the neighborhood relationship. Consequently, the opportunity of the accidental contact increases, and it weakens the influence of the original network structure.

However, when the D2D mobility model is replaced by the real-world vehicle mobility model, which is derived from the GPS traces dataset in Italy [17], the performance of all the strength based algorithms are approximate and keep low. This may be caused by the sparse distribution of vehicles and their movement restrictions. Its mobility model is shown in Fig. 3. In fact, the location of each node updates every 30 seconds. We record the nodes' mobility trajectory with a 900-second interval. Their mobility trajectory is connected by a red line for each node. We can conclude that most of vehicles are confined to their own small circle and the distances among them are relatively far.

V. Asymmetry Analysis

In order to clarify the essence of asymmetric strengths in terms of their superior performances, we analyze the asymmetry characteristic by means both of the graph structure (Section V-A), and of the experiment statistics (Section V-B).



(a) Value and partial strength (b) Asymmetric and symmetric value strength

(c) Asymmetric and symmetric partial strength (d) Mobile networks

Fig. 5. Asymmetry analysis: a perspective of the graph structure

A. Asymmetry Analysis in terms of Graph Structures

In this subsection, we first explain the reason why different algorithms have various performances from a lateral comparison. Then, from a longitudinal comparison, the superiority of asymmetric algorithm is analyzed. Finally, the information diffusion mechanism in mobile networks can be specified.

The four topologies in Fig. 5 are utilized in this subsection, where the color of white, gray and blue indicate the state of I_0, I_1 and I_2, respectively. The red solid line in the node a to b or c means that the forwarding node a will select the next relay node between b and c. In the 5 (d), the dotted circles are the movement range of the node and the dotted lines means that the two points may establish a connection.

Previous work pointed out that the partial strength and the value strength performed better than the tie strength with the consideration of the power law distribution of node degree. Specifically, the denominator, i.e., $C_a \cup C_b$, in the Eq. (3) and (4) are almost C_a when $C_a \gg C_b$. Besides, from a lateral comparison, we find asymmetric strong value performs better than asymmetric weak partial both in static and mobile network. Hence, we first explore the partial strength and the value strength in both asymmetric and symmetric forms. From the perspective of the graph, these can be explained as follows. As shown in Fig. 5 (a), the source a will make the choice of the next hop node between b and c. Obviously, the node h and g have received the information when the node a forward the information. Therefore, regardless of other special extension topology, the increase of the information diffusion coverage rate of the node c will be no greater than the node b. This is because the node d, e and f have not been informed, which could be utilized to further improve the information coverage rate if the node b is selected. Calculating the value strength and partial strength, respectively, we can achieve the point. Considering the partial strength, we have $S_{SP}(b \to a) = 1/4$, $S_{SP}(c \to a) = 3/4$, $S_{AP}(b \to a) = 1/4$ as well as $S_{AP}(c \to a) = 3/3$, and the node c seems to be better than the node b in both two definitions. However, the results are $S_{SV}(b \to a) = 3/4$, $S_{SV}(c \to a) = 0/3$, $S_{AV}(b \to a) = 3/4$ and $S_{AV}(c \to a) = 0/4$. In these two cases, the node b is better than node c.

The second graph in Fig. 5 depicts why asymmetric strong value is superior versus symmetric strong value. Numerically, the results are $S_{SV}(b \to a) = 1/4$, $S_{SV}(c \to a) = 1/2$, $S_{AV}(b \to a) = 1/4$ and $S_{AV}(c \to a) = 1/4$, respectively. The node b is inferior to the node c in symmetric way. However, this may be inappropriate. Although node b and node c are with the same new neighbor, i.e., node e and node d, the node b has more possibility to make the information diffusion continuing. If the node c wins the election, then node d will be the next information source. Similarly, without consideration of other special extension topology, the information diffusion will be trapped, as there is no next node can be chosen as the next information source.

Then, the comparison between asymmetric and symmetric partial strength is similar to the difference between the asymmetric and symmetric value strength. What should be highlighted here is the fact that there still exist a doubt why the "weak" but not "strong". Considering the graph in Fig. 5 (c), the results are $S_{AP}(b \to a) = 2/2$ and $S_{AP}(c \to a) = 1/4$, respectively. In terms of the asymmetric strong partial, the node b wins the node c. However, the node c is more important than the node b to a certain extent, because the node c connects the three new nodes. The selection results will be reversed in terms of the asymmetric weak partial strength.

At last, we know that the dynamic network is often composed of disconnected pieces, such as the graph in Fig. 5 (d), where information diffusion will not be too far. However, on one hand, it is the mobility that makes the information diffusion becoming possible. When a node moves towards another node, the information may be delivered as shown in the red dashed line. Contrarily, if the distance is too far to establish a connection, the information transmission will be blocked. That is why the information coverage rate is relatively low in the Italy vehicular network. On the other hand, the mobility results in a frequent changes in the graph structure and weakens the differences among the algorithms.

B. Asymmetry Analysis in terms of Experiment Statistics

In this subsection, we count the number of the times of each algorithm falling into information local traps under 50 independent tests based on the Flickr dataset.

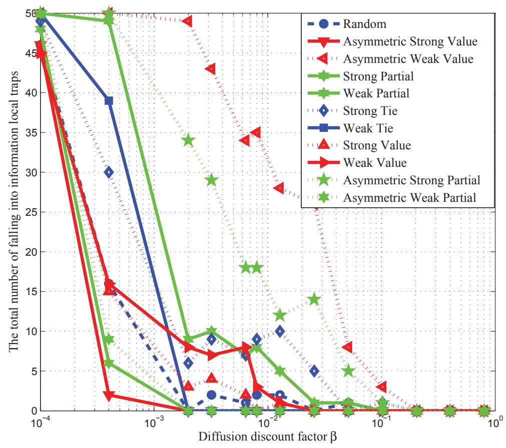


Fig. 6. The total number of times of falling into information local traps.

The Fig. 6 depicts the total times of falling into information local traps of each algorithm versus different diffusion discount factor β. We can conclude that the algorithm with good performance is often accompanied by a few number of times of falling into information local trap.

VI. Conclusion

In a nutshell, relying on the characteristics analysis of the partial strength and value strength, we consider their asymmetric and symmetric normalization forms. The results show that the asymmetric strength are capable of improving the coverage of information diffusion in socially-aware mobile networks. Moreover, we analyze the essence of asymmetry and its influence on information diffusion based on the graph structures as well as the experiment statistics. Simulation results conducted both on the static network and on the mobile network show the performances of different strengths aided information diffusion scheme. Our work may be beneficial in terms both of facilitating useful information transmitting as well as of preventing malicious information broadcasting in socially-aware mobile networks, such as the viral marketing, the public opinion control, etc.

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